Useful Formulae in Signal Processing

Autocorrelation Function

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)\, x(t+\tau) dt$$

Cross-correlation Function

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)\, y(t+\tau)\, dt$$

Power Spectral Density

$$G_{xx}(f) = \frac{1}{B_E} \psi_x^2(f, B_E)$$

$$S_{xx}(f) = \frac{1}{T} X(f) X^*(f)$$

$$G_{xx}(f) = \frac{2}{T} X(f) X^*(f)$$

Wiener-Khintchine Relationship

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) \exp(-j2\pi f\tau)\, d\tau$$

$$G_x(f) = 4\int_0^{\infty} R_x(\tau) \cos(2\pi f\tau)\, d\tau$$

$$R_x(\tau) = \int_0^{\infty} G_x(f) \cos(2\pi f\tau)\, df$$

Cross Spectral Density Function

$$G_{xy}(f) = \frac{2}{T} X^*(f)\, Y(f)$$

$$\left|G_{xy}(f)\right| = \sqrt{C_{xy}^2(f) + Q_{xy}^2(f)}$$

$$\theta_{xy}(f) = \tan^{-1}\left|\frac{Q_{xy}(f)}{C_{xy}(f)}\right|$$

$$G_{xy}(f) = 2\int_{-\infty}^{\infty} R_{xy}(\tau) \exp(-j2\pi f\tau)\, d\tau$$

Frequency Response Functions

$$H_0(f) = \frac{G_{xy}(f)}{G_{xx}(f)} = \frac{G_{yy}(f)}{G_{yx}(f)}$$

$$H_1(f) = \frac{H_0(f)}{1 + \frac{G_{mm}(f)}{G_{uu}(f)}}$$

$$H_2(f) = H_0(f)\left[1 + \frac{G_{nn}(f)}{G_{vv}(f)}\right]$$

Coherence Function

$$\hat{\gamma}_{xy}^2(f) = \frac{\left|\hat{G}_{xy}(f)\right|^2}{\hat{G}_x(f)\hat{G}_y(f)}$$

Time delay bias error

$$\hat{\gamma}_{xy}^2(f) \approx \left(1 - \frac{\tau_1}{T}\right)^2 \gamma_{xy}^2(f)$$

Convolution Integral

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau) d\tau$$

Coherent Output Power Function

$$\hat{G}_{vv}(f) = \hat{\gamma}_{xy}^2(f).\hat{G}_{yy}(f)$$

Incoherent Output Power Function

$$\hat{G}_{nn}(f) = \left[1 - \hat{\gamma}_{xy}^2(f)\right]\hat{G}_{yy}(f)$$

All terms have their usual meaning